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Seasonal Adjustment Using Structural Time Series Models: An Application and a Comparison With the Census X-11 Method

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This article makes the method of seasonal adjustment operational using suitable structural time series models (STM). This so-called STM method is applied to several relevant Dutch macroeconomic quarterly and monthly time series. The results are compared with those of the Census X-11 method using several formal criteria as yardsticks. The STM method proves to compete well with the Census X-11 method in this respect.

KEY WORDS: Kalman filter; Model decomposition.

1. INTRODUCTION

This article reports on seasonal adjustment of a number of Dutch macroeconomic quarterly and monthly series using structural time series models (Engle 1978; Harvey and Todd 1983). The study consists of three parts. First, the method of seasonal adjustment is made operational for practical purposes. Although some empirical examples of structural models for seasonal time series are given by the authors just mentioned and by, for example, Gersch and Kitagawa (1983) and Harvey (1984), the use of these models for seasonal adjustment is, up to now, still in the laboratory phase. As far as we know, structural time series models are not yet used for seasonal adjustment on a regular basis—for example, by statistical agencies. Our aim is the design of a prototype of an operational method that exploits results from the literature rather than uses its own new theoretical developments. Our method, which we call the structural time series model (STM) method, is discussed in Section 2. Second, the STM method is applied to selected time series. The outcomes are presented in Section 3. Third, following earlier comparative analyses of seasonal adjustment methods (Den Butter, Coenen, and Van de Gevel 1985; Fase, Koning, and Volgenant 1973), the STM method is compared with the Census X-11 method using several formal criteria and on a theoretical level. This comparison is set out in Section 4. Finally, Section 5 gives the conclusions.

2. THE STM METHOD

2.1 Selected Models

The usual formulation of a seasonal time series structure is as follows: $y_t = T_t + S_t + I_t$, where y_t is the observation of the time series y at time t, T is the trend(-cycle) component, S is the seasonal component, and I is the irregular component.

For an operational model-based method of seasonal adjustment, a relatively small set of standard models that are suitable in describing a great variety of economic time series is needed [see Sims (1985) and the procedure in the automatic option of the X-11-ARIMA program (Dagum 1980)]. Here, the following trend specification is assumed:

$$T_t = T_{t-1} + T^*_{t-1} + v_t$$

$$T^*_t = T^*_{t-1} + v^*_t.$$

For the seasonal component, the following two alternatives are selected:

$$S_t = -\sum_{i=1}^{s-1} S_{t-i} + w_t \tag{1}$$

and

$$S_t = S_{t-s} - \frac{1}{s}\sum_{i=1}^{s} S_{t-i} + w_t, \tag{2}$$

where s is the seasonal period ($s = 4$ for quarterly and $s = 12$ for monthly data) and v_t, v^*_t, and w_t are independent white noise (just as I_t is). By combining the trend and the seasonal components (1) and (2), two alternative structural time series models—M1 and M2, respectively—result. The parameters to be estimated from these models are the variances of the noises (σ^2_I, σ^2_v, $\sigma^2_{v^*}$, and σ^2_w, respectively).

The specification for the trend is taken from the literature (e.g., see Harvey 1984). This specification describes a trend whose change constitutes a random walk.

The specification is suitable for second-order nonstationary series. In case $\sigma^2_{v^*} = 0$, the trend follows a random walk with deterministic drift T^*. The specification of the seasonal component (1) is also taken from the literature mentioned previously. This specification implies that in each period the sum of the seasonal components per year is white noise; $\Sigma_{i=0}^{s-1} S_{t-i} = w_t$ for all t. In other words, the expectation of the sum of the seasonal components is always 0, but the sum itself may deviate from the expectation; the specification thus describes a shifting seasonal pattern (if $\sigma^2_w \neq 0$) in which the deviations have a constant variance. The specification of the seasonal component in (2) can be rewritten as $(1 - \rho_s L) \Sigma_{i=0}^{s-1} S_{t-i} = w_t$ when $\rho_s = (s - 1)/s$, which shows that in each period the sum of the seasonal components per year follows a first-order autoregressive [AR(1)] process with parameter ρ_s. Of course, both (1) and (2) are special cases of the general AR(1) model $(1 - \rho L) \Sigma_{i=0}^{s-1} S_{t-i} = w_t$, where $\rho = 0$ in (1) and $\rho = (s - 1)/s$ in (2). The STM method selects from the two preceding specifications with fixed ρ.

2.2 Preliminary Model Identification

The decision whether the seasonal adjustment is to be additive or multiplicative is made by inspecting the proportionality of preliminary estimates of the trend and the seasonal components for each series to be adjusted. This ad hoc selection is common practice in seasonal adjustment at the Nederlandsche Bank and may, in a later version of the method, be replaced by a more formal procedure.

The most important identification problem of the STM method is to determine which of the two models, M1 and M2, is the most suitable one. We have tried a formal identification procedure by estimating an unrestricted (autoregressive moving average) ARMA(1, s + 2) model for the series $\Delta\Delta_s y^*$ (where $y^* = y$ in case of additive and $y^* = \log y$ in case of multiplicative adjustment). It yields a consistent, albeit not fully efficient, estimate of the autoregressive parameter ρ. To select the adequate model, we have tested the hypotheses that $\rho = 0$ and that $\rho = (s - 1)/s$. This test did not lead to a useful selection between both models, however, because the test failed to discriminate between the models in 4 out of 23 cases and rejected both models in 7 out of 23 cases. Moreover, in 3 cases the estimate of ρ was almost equal to unity. It implies a common root in the trend and seasonal component. This is an awkward property for seasonal decomposition because changes in the seasonal pattern get mixed up with changes in the trend (Harvey and Todd 1983).

The sample autocorrelation r_s from the preliminary seasonal components of y^* has been used as an informal identification criterion instead. By rule of thumb, we selected M1 for the quarterly series if r_4 is smaller than .45 and M2 if r_4 is greater than .55, both specifications being considered in case r_4 lies between .45 and .55. For the monthly series, the corresponding values are .70 and .90, respectively. As yet, this procedure gives a preliminary identification of the models only. After the estimation stage, we apply diagnostic tests on model adequacy, which leads to a final selection of the most adequate model.

2.3 Estimation Method

For estimating the parameters of the models M1 and M2, we follow Harvey and Todd (1983) by maximizing numerically the likelihood function in the time domain. For this reason, the models have been put in the state-space form. The signal-to-noise ratios σ^2_v/σ^2_I, $\sigma^2_{v^*}/\sigma^2_I$, and σ^2_w/σ^2_I are estimated by maximizing the logarithm of the likelihood function concentrated to σ^2_I. The estimation of σ^2_I uses the one-period-ahead forecast errors of the series y^* that are calculated via the Kalman filter.

The application of the Kalman filter needs values $\vec{\Theta}_{0|0}$ and $\overline{P}_{0|0}$ for the state vector $\vec{\Theta}_t$ and $\text{var}(\vec{\Theta}_t)/\sigma^2_I$ when $t = 0$. We have chosen to apply a so-called diffuse prior, which means that $\overline{P}_{0|0}$ has been given very large values for the elements on the main diagonal. The result is that the starting vector $\vec{\Theta}_{0|0}$ has a very small effect on the subsequent estimates of $\vec{\Theta}_t$, and therefore its (arbitrary) choice is of little importance. It can be shown that, if $\overline{P}_{0|0}$ is equal to $k.I_n$ with $k \to \infty$, the Kalman filter results tend to those obtained by using starting values that are computed from the first n observations by means of generalized least squares, assuming that these observations are fixed (Harvey and Peters 1984). The choice for $P_{0|0}$ as a diagonal matrix did, however, for some series, yield estimates of the seasonal components with the same sign for all periods $n + 1$ up to and including $n + s$. For this reason, we have used the first $2n$ observations as the initial period. In maximizing the likelihood function, the observations from this initial period are not taken into consideration.

As the preceding procedure proves to be rather computer time consuming, we mention three alternative estimation methods that are asymptotically equivalent to the method used by us. Harvey and Peters (1984) described both estimation in the time domain via the EM algorithm (Dempster, Laird, and Rubin 1977) and estimation in the frequency domain. As a third alternative, F. C. Palm, in a personal communication, suggested estimating the unrestricted reduced-form specification of the structural time series model to obtain consistent parameter estimates and then, in a second step, applying asymptotic least squares (see Gouriéroux, Monfort, and Trognon 1982; Kodde and Palm 1985), taking the nonlinear restrictions on the parameters into account. The estimation of the parameter ρ in the formal identification procedure referred to in Section 2.2—which failed to yield useful results—is, in fact, identical to the first part of this suggestion.

2.4 Decomposition

Given the estimates for the variances of the noise in the model (denoted by σ_I^2, σ_v^2, $\sigma_{v^*}^2$, and σ_w^2) the Kalman filter provides estimates of the trend and seasonal component of y^*, since these are elements of the state vector. The irregular component is found as the residual.

For estimation of the variances of the noise, all observations are used. Therefore, it seems logical that in the decomposition all observations also are exploited and that the seasonal adjustment uses the so-called smoothed Kalman filter estimates that are computed by the fixed interval algorithm (Anderson and Moore 1979; Harvey 1981). They theoretically yield the best approximation to the trend and seasonal components, given the model and its parameters, since they have minimum mean squared errors conditional on all observations used. In this respect, we note that an equivalent method of decomposition was proposed by Cleveland and Tiao (1976) and Bell (1984).

When an additive adjustment is made, the estimates for the trend and seasonal components immediately give the seasonally adjusted series $y^{sa} = T + I$. In the case of a multiplicative adjustment, the seasonally adjusted series is calculated as $y^{sa} = \exp(T + I)$ and the seasonal component as $y - y^{sa} = (\exp(S) - 1)y/\exp(S)$.

3. APPLICATION OF THE STM METHOD

3.1 Series Examined

To illustrate the application of the method set out previously, 14 quarterly series for the period 1970:I–1983:IV and nine monthly series for the period 1970:1–1983:12 are selected. Most of them are time series of practical importance in Dutch macroeconomic policy analysis and were already studied earlier for different periods at The Nederlandsche Bank by Fase et al. (1973) and Den Butter et al. (1985). The sources of the data are given in Table 1.

3.2 Application to Quarterly Series

Table 2 gives the identification and estimation results for the 14 quarterly series. Ten series show some pro-

Table 1. Description and Sources of the Data

Data	Unit	Source
Quarterly series (period 1970:I–1985:IV)		
1. Net foreign assets	Millions of guilders	DNB, KB
2. Floating debt of central government and local authorities	Millions of guilders	DNB, KB
3. Net money-creating operations	Millions of guilders	DNB, KB
4. Miscellaneous assets and liabilities with banks and giro institutions	Millions of guilders	DNB, KB
5. Domestic money stock (M2)	Millions of guilders	DNB, KB
6. Financial assets, private sector	Millions of guilders	DNB, KB
7. Average daily output of manufacturing industry at constant prices	Index 1980 = 100	CBS1
8. Financial deficit of central government and local authorities	Millions of guilders	DNB, KB
9. Total unemployment	Number of man-years	SoZa
10. Gross hourly wage rates, industry	Index 1972 = 100	CBS2
11. Total imports of goods (CIF), services and earnings paid	Millions of guilders	DNB, KB
12. Total exports of goods (FOB) and services and earnings received	Millions of guilders	DNB, KB
13. Consumers' expenditure	Index 1975 = 100	CBS3
14. Gross industrial investment (excluding housing and stocks)	1980 prices	DNB, KC
Monthly series (period 1970:1–1985:12)		
1. Net foreign assets	Millions of guilders	DNB, KB
2. Floating debt of central government and local authorities	Millions of guilders	DNB, KB
3. Net money-creating operations	Millions of guilders	DNB, KB
4. Miscellaneous assets and liabilities with banks and giro institutions	Millions of guilders	DNB, KB
5. Domestic money stock (M2)	Millions of guilders	DNB, KB
6. Average daily output of manufacturing industry at constant prices	Index 1975 = 100	CBS1
7. Male unemployment	Number of persons	SoZa
8. Total unemployment excluding construction industry	Number of persons	SoZa
9. Unemployment construction industry	Number of persons	SoZa

NOTE: Abbreviations for sources are as follows: CBS1—Central Bureau of Statistics, Maandstatistiek voor de Industrie (Monthly Industrial Bulletin); CBS2—Maandschrift (Monthly Bulletin); CBS3—Statistisch Bulletin (Statistical Bulletin); DNB, KB—De Nederlandsche Bank N.V., Quarterly Bulletin; DNB, KC—De Nederlandsche Bank N.V. (1982 and 1986), Kwartaalconfrontatie van Middelen en Bestedingen (Quarterly National Income and Expenditure Accounts); SoZa—Ministry of Social Services and Employment, Maandverslag Arbeidsmarkt (Monthly Labour Market Report).

Table 2. Identification, Estimates, and Diagnostics of the Quarterly Series

Series	M or A[a]	Specifications	Parameter estimates[c] σ_ι^2	σ_v^2	$\sigma_{v^*}^2$	σ_w^2	Diagnostics[d] LB	H	AIC	Final selection of model
1. Net foreign assets	M	M2	.013	1.224	.549	.040	4.32	.30		M2
2. Public authority floating debt	M	(a) M1	.093	7.056	.405	.972	15.89	3.25	−51.47	M2
		(b) M2	.161	5.018	.446	2.643	12.23	3.10	−51.72	
3. Net money-creating operations	M	M2	.007	.193	.830	.002	14.19	1.10		M2
4. Miscellaneous items	M	M1	.907	9.107	.000	.000	14.57	1.18		M1
5. Domestic money stock	M	M2	.006	.184	.097	.013	12.52	2.63		M2
6. Private-sector financial assets	M	M1	.003	.004	.002	.000	14.48	1.17		M1
7. Manufacturing output	M	(a) M1	.038	.431	.000	.009	13.09	2.44	−202.51	M1
		(b) M2	.063	.424	.002	.014	15.09	2.44	−190.53	
8. Public authority financial deficit	A[b]	M1	4.073	96.696	2.686	403.526	11.51	.37		M1
9. Unemployment	A	M2	.001	.099	.036	.005	4.96	.43		M2
10. Wage-rate index	A	M2	.089	3.106	.074	.002	4.83	.51		M2
11. Imports	M	M2	.161	1.024	.092	.031	7.84	2.31		M2
12. Exports	M	(a) M1	.016	1.759	.004	.006	3.28	1.46	−148.30	M1
		(b) M2	.057	1.776	.000	.007	4.38	2.00	−140.62	
13. Consumers' expenditure	A	M1	.414	.145	.057	.032	8.90	1.22		M1
14. Gross investment	A	M1	76.061	133.588	.015	1.011	10.12	1.68		M1

[a] M means multiplicative; A is additive.

[b] The series assumes negative values.

[c] For presentation purposes, the parameter values were multiplied by 10^3 in the case of multiplicative seasonal adjustment; the parameter values of series 8, 9, and 14, for which additive adjustment was carried out, were multiplied by 10^{-3}.

[d] The explanation of diagnostics is as follows: LB—Ljung–Box portmanteau test for white noise with asymptotic χ^2 (12) distributed with a 95% critical value of 21.0; H—Harvey's (1985) test for heteroscedasticity with $F(15, 15)$ distributed with (.35, 2.86) as the 95% confidence region; AIC—Akaike's information criterion, mentioned only if more than one model is estimated for one series (a lower criterion value implies a better model for a certain series).

portionality between the seasonal component and the trend. In these cases, a multiplicative adjustment is made, except for the public authority financial deficit, because this series assumes negative values, so eventually for 5 series an additive adjustment is chosen. For public authority floating debt, manufacturing output (at constant prices), and the value of exports, the identification of the seasonal component using our rule of thumb does not lead to a unique choice. Therefore, in these instances both specifications are examined. The last column in Table 2 gives the models that, after diagnostic testing, have finally been selected in the identification procedure. The two test statistics LB and H relate to the irregular component. We note that Harvey (1985) used the same statistics. Both have been calculated on the basis of the standardized one-period-ahead forecast errors, using the Kalman filter.

With no exceptions, the LB test statistic indicates suitable model specifications. On the other hand, Harvey's H test for heteroscedasticity points to misspecification of 3 out of the 17 models that have been estimated. Necessarily, since the present version of the STM method considers specifications M1 and M2 only, the unsatisfactory results of M2 for net foreign assets and both M1 and M2 for public authority floating debt have to be accepted. For those series in which two models have been estimated and no selection could be made, Akaike's information criterion (AIC) is used as a yardstick by selecting the specification with the lowest AIC. Incidentally, the estimates only show small differences between the various specifications for most series. In these cases, the selection procedure does not have much influence on the final seasonal adjustment.

3.3 Application to Monthly Series

The results of the identification and estimation procedure are shown in Table 3. Almost all series considered show proportionality between the seasonal component and the trend, indicating a multiplicative adjustment. The sole exception is male unemployment, where an additive adjustment is applied. For most of the monthly series, the rule-of-thumb estimate of the sample autocorrelation r_{12} estimate is higher than .9, which points to model M2. For several series, however, model M1 also is estimated because of unsatisfactory results for the diagnostic checks, particularly LB. The parameter estimates demonstrate that, as with the quarterly series, in general a greater part of the stochastics is attributed to shifts in the trend or seasonal component than to the irregular component.

In contrast to the quarterly series, the LB test statistic indicates that in several cases the residuals of the monthly series are not white noise. Model M1 has been selected for unemployment in the construction industry because of a slightly "better" value for LB. In addition, model M1 has also been chosen for miscellaneous items because, according to Harvey's H test, this model is adequate, whereas M2 is not. The models for net foreign assets and male unemployment have been accepted in spite of their inadequacy indicated by Harvey's H test; for these series, the test leads to rejection of both models considered. For the other relevant series the lowest value of AIC determines the choice of specification. Quite remarkably, the selected specification for public authority floating debt, net money-creating operations, and industrial unemployment appears to be model M1, which in the first instance does not follow

Table 3. Identification, Estimates, and Diagnostics of the Monthly Series

Series	M or A[a]	Specifications[b]	Parameter estimates[c]				Diagnostics[d]			Final selection of model
			σ_ι^2	σ_v^2	$\sigma_{v'}^2$	σ_w^2	LB	H	AIC	
1. Net foreign assets	M	M2	.134	.659	.019	.015	26.94	.26		M2
2. Public authority floating debt	M	(a) M1*	.231	5.610	.003	.000	102.77	.88	−297.22	M1
		(b) M2	.228	5.331	.005	.068	115.15	.95	−288.96	
3. Net money-creating operations	M	(a) M1*	.013	.108	.044	.001	35.52	.59	−746.91	M1
		(b) M2	.015	.097	.047	.004	36.20	.59	−733.99	
4. Miscellaneous items	M	(a) M1	3.836	3.852	.000	.045	37.69	1.82	−219.69	M1
		(b) M2	1.406	6.291	.000	.732	38.43	2.16	−206.26	
5. Domestic money stock	M	M2	.014	.100	.002	.014	56.68	.89		M2
6. Manufacturing output	M	(a) M1	.119	.141	.000	.084	42.40	1.25	−612.43	M1
		(b) M2	.056	.221	.000	.198	51.07	.98	−592.03	
7. Male unemployment	A	M2	755.2	1976.0	1339.7	1679.4	51.04	.49		M2
8. Industrial unemployment	M	(a) M1*	.000	.000	.545	.010	388.87	1.08	−595.62	M1
		(b) M2	.000	.001	.396	.050	519.34	1.43	−595.45	
9. Unemployment construction industry		(a) M1*	.178	.212	6.638	.058	167.06	1.16	−236.60	M1
		(b) M2	.143	.239	5.367	.349	178.12	1.30	−237.48	

[a] M means multiplicative; A is additive.

[b] The specifications marked with an asterisk have been estimated because of the poor results with respect to first-order or higher-order autocorrelations obtained in estimating the specifications identified in the preliminary procedure.

[c] For presentation purposes, the parameter values were multiplied by 10^3 in the case of multiplicative seasonal adjustment.

[d] The explanation of diagnostics is as follows: LB—Ljung-Box portmanteau test for white noise with asymptotic χ^2 (36) distributed with a 95% critical value of 51.0; H—Harvey's (1985) test for heteroscedasticity, with $F(45, 45)$ distributed with (.53, 1.88) as the 95% confidence region; AIC—Akaike's information criterion, mentioned only if more than one model is estimated for one series (a lower criterion value implies a better model for a certain series).

from the identification procedure. Incidentally, the differences between these alternative specifications are extremely small.

4. COMPARISON WITH CENSUS X-11

4.1 Seasonal Adjustment Using Census X-11

The STM method is compared with the Census X-11 method (see Shiskin, Young, and Musgrave 1967) to assess its quality as a useful method for seasonal adjustment in practice. In principle, Census X-11 is a heuristic method, which separates the trend-cycle component from the seasonal and irregular components by repeated application of weighted moving averages. It enjoys its worldwide popularity as a seasonal adjustment method mainly because of its wide applicability and the flexibility with which shifts in the seasonal pattern can be described. Earlier comparative analyses of seasonal adjustment methods (Den Butter et al. 1985; Fase et al. 1973) show Census X-11 to be the best method in this respect. Hence this method has been used at the Nederlandsche Bank for nearly 17 years.

4.2 Practical Comparison by Formal Criteria

For the comparative analysis, all quarterly and monthly series considered are seasonally adjusted with Census X-11 in the conventional way. The multiplicative variant is applied to the series for which the STM method also estimates multiplicative models. Mutatis mutandis, the additive variant of Census X-11 has been used. The observation period corresponds with the estimation period of the structural models—thus excluding the initial period (which differs slightly per model).

The comparative study by Fase et al. (1973) provided formal criteria by which seasonal adjustment methods can be judged. Although their use may be open to discussion from a theoretical point of view, these criteria are considered to be a well-established means of selection by practitioners of seasonal adjustment at statistical agencies. In the present study, these criteria for the STM method are based on the smoothed estimates of the trend, seasonal, and irregular components and not on the one-period-ahead forecasts such as the diagnostics of Sections 3.2 and 3.3. The reason is that these smoothed estimates are actually used in the seasonal adjustment. The results are given in Table 4 (the Appendix provides the definitions of the criteria).

4.2.1 Mean Absolute Percentage Change. This measure indicates the smoothness of the seasonally adjusted series in relation to the original series. Smoothness is considered an attractive quality of seasonally adjusted series because it simplifies policy analysis based on these series. Table 4 shows that seasonal adjustment by means of Census X-11 and by means of the STM method do not differ much in smoothness, although the lowest values are found more often with Census X-11 than with STM, notably in the case of the monthly series.

4.2.2 Orthogonality. This criterion values the correlation between the seasonal component and the seasonally adjusted series. The ideal situation is zero correlation and hence perfect orthogonality. As shown in Table 4, this situation is reasonably well met for all series by both adjustment methods, except for the monthly series of unemployment in the construction industry. Although most of the lowest criterion value (in absolute terms) are found with Census X-11, this criterion does not clearly point to the superiority of either method. We note that in theory the STM method yields orthogonal components, since these are modeled as such in specifications M1 and M2. The estimated components only approximate orthogonality, however (Maravall 1987).

Table 4. Comparison of the STM Method With Census X-11

			Judgment criteria									
			Mean absolute percentage change		*Orthogonality*		*Idempotency*		*Stability all common years*		*Stability latest common year*	
Data	*Adjustment*	*Model*	*X-11*	*STM*	*X-11*	*STM*	*X-11*	*STM*	*X-11*	*STM*	*X-11*	*STM*
			Quarterly series									
1. Net foreign assets	M	M2	4.92	4.99	−.04	.00	.18	.00	.19	.27	.85	.74
2. Public authority floating debt	M	M2	7.87	6.34	−.03	−.01	1.15	.10	.34	.55	1.50	2.11
3. Net money-creating operations	M	M2	3.92	3.88	−.04	.03	.18	.00	.05	.07	.23	.12
4. Miscellaneous items	M	M1	8.52	9.04	−.02	.04	.50	.00	.59	1.12	2.11	2.30
5. Domestic money stock	M	M2	2.65	2.63	−.04	−.03	.10	.00	.03	.03	.15	.12
6. Private-sector financial assets	M	M1	2.88	2.88	−.04	−.04	.02	.00	.02	.02	.04	.06
7. Manufacturing output	M	M1	1.68	1.67	.04	.05	.10	.00	.02	.03	.12	.12
8. Public authority financial deficit	A	M1	*	*	−.00	−.04	*	*	4.01	1.38	9.24	7.45
9. Unemployment	A	M2	5.93	5.97	−.02	−.04	.37	.00	.06	.13	.17	.21
10. Wage-rate index	A	M2	1.93	1.95	−.05	−.08	.04	.00	.02	.02	.10	.02
11. Imports	M	M2	3.48	3.53	.05	.10	.21	.00	.12	.10	.51	.27
12. Exports	M	M1	3.71	3.91	.01	.02	.19	.00	.10	.21	.37	.49
13. Consumers' expenditure	A	M1	.06	.95	.02	.00	.07	.00	.02	.05	.11	.18
14. Gross investment	A	M1	4.36	4.74	.04	.01	.23	.03	.14	.27	.58	.37
			Monthly series									
1. Net foreign assets	M	M2	2.49	2.59	−.01	−.06	.27	.01	.28	.45	1.14	.83
2. Public authority floating debt	M	M1	5.00	5.39	−.05	−.12	1.15	.00	.71	.77	2.57	.83
3. Net money-creating operations	M	M1	1.59	1.60	−.03	.00	.18	.00	.08	.11	.35	.15
4. Miscellaneous items	M	M1	6.40	7.33	.01	.02	.72	.00	.40	.78	1.86	1.50
5. Domestic money stock	M	M2	1.20	1.10	−.04	−.03	.14	.01	.06	.13	.22	.31
6. Manufacturing output	M	M1	1.80	1.11	.03	.03	.24	.01	.06	.07	.28	.28
7. Male unemployment	A	M2	2.16	2.03	−.00	−.01	.42	.01	.06	.18	.16	.16
8. Industrial unemployment	M	M1	2.03	2.34	.05	−.06	.53	.01	.23	.39	.91	.54
9. Unemployment, construction industry	M	M1	5.46	6.51	−.23	−.34	1.26	.00	.93	1.27	3.16	1.28

* Calculation of criterion value is not meaningful, since some observations contained in the series are almost equal to 0.

4.2.3 Idempotency. This criterion shows to what extent the seasonally adjusted series still contains seasonal fluctuations. A seasonal adjustment method is called idempotent if application of that method to the seasonally adjusted series yields exactly the same series. The results in Table 4 show that the STM method is virtually fully idempotent and (somewhat to our surprise in view of the results on orthogonality) much better on this score than Census X-11.

4.2.4 Stability. This criterion shows to what extent seasonal adjustment of old data changes when new data are added. Policy analysis is usually based on recent data; therefore, stability of the figures is very desirable and an important criterion in seasonal adjustment. Table 4 presents two different measures for stability. Both relate to the successive addition of two years (1984 and 1985) to the data, by which an annual addition of new data is simulated. The first measure gives the mean absolute percentage change in the seasonally adjusted data over all the years both series have in common when new data are added. The second measure concerns the latest year in common—that is, 1983 when adding 1984 and 1984 when adding 1985. For both measures, a zero value means full stability.

The results of Table 4 show that, with regard to the criterion of stability over all common years, Census X-11 is more satisfactory for the vast majority of the series than the STM method. Note, however, that the STM method uses new parameter estimates and hence new smoothed estimates for the components over the whole observation period when adding new data. On the other hand, with Census X-11, figures relating back to more than four years do not change any more when new data are added. This is a pragmatic rule of Census X-11, which could also be applied to the STM method. It explains the relative good performance of Census X-11 on this criterion.

According to the second criterion, the STM method, on the whole, shows a greater stability than Census X-11. In sum, this means that the addition of new data affects the seasonal adjustment more over the whole observation period when using the STM method than when using Census X-11, but for recent data, the changes are greater with Census X-11 than with the STM method.

4.3 Theoretical Comparison of the STM Method and Census X-11

The STM method can be compared with Census X-11 on a theoretical level rather than by formal criteria. Cleveland (1972) approximated the monthly Census X-11 procedure by the structural time series model with a reduced form (autoregressive integrated moving average) ARIMA(0, 1, 14)*(0, 1, 0)$_{12}$ model for the original series y_t^*. In this model, out of 15 parameters, only the variance of the noise is to be estimated freely. On the other hand, model M1 corresponds in its reduced form to an ARIMA(0, 1, 13)*(0, 1, 0)$_{12}$ model with 10 restrictions on 14 parameters. Hence, as compared to M1, the reduced form ARIMA model of Census X-11 contains only 1 extra moving average parameter, and its parameter values are bound to considerably more restrictions. Therefore, as Maravall (1985) showed, by selecting appropriate parameter values model M1 may very well approximate the theoretical autocorrelations of $\Delta\Delta_{12}y_t$ according to Census X-11. Hence, from this theoretical point of view, the STM method would almost always be superior to Census X-11, since the comparison between both methods for individual series by means of a nonnested hypothesis test would not, in most cases, be in favor of Census X-11. Probably the same holds for the quarterly versions of the STM method and Census X-11, although the reduced form ARIMA model of the quarterly version of Census X-11 may be difficult to determine.

Even if the reduced-form models are statistically equivalent for a particular series, however, the corresponding decompositions differ among each other. Both reduced-form ARIMA models allow for an infinite number of decompositions as a consequence of the underidentification of the individual components. Therefore, a comparison of the decompositions of the STM method and Census X-11—for example, by analyzing the filter coefficients—can only be made by means of specific criteria. The formal criteria that we used refer to the practitioners' notion of seasonal adjustment [for a discussion, see Fase et al. (1973) and the references therein]. We appreciate that such a set of criteria can be easily extended and is open to discussion from a theoretical point of view. A decomposition method could be chosen that, by definition, satisfies one of the formal criteria. As a matter of fact, much attention has been focused recently on the so-called canonical decomposition of a (reduced-form) time series model into its structural components (e.g., see Hillmer and Tiao 1982; Maravall 1985; Maravall and Pierce 1987; Pierce 1978). For a practical method of seasonal adjustment, however, we would not favor this decomposition, which maximizes the variance of the irregular component, because it would lead to a rather unstable seasonally adjusted series (see Maravall 1986).

4.4 Evaluation

The inequality coefficients in Table 5 give the size of the differences between the seasonal components of Census X-11 and the STM method. In the case of the quarterly series, the greatest differences occur with net money-creating operations in which—as we have learned from inspection of the time profile of the seasonal components—at the end of the observation period the shift in the seasonal component is much larger

Table 5. Evaluation of Differences Between the Seasonal Components According to Census X-11 and the STM Method

Data	Inequality coefficient	Mean size		Mean change	
		X-11	STM	X-11	STM
Quarterly series					
1. Net foreign assets	.18	1,113	792	.23	.24
2. Public authority floating debt	.13	6,370	6,252	.14	.18
3. Net money-creating operations	.29	1,717	1,829	.22	.12
4. Miscellaneous items	.09	2,007	1,971	.19	.18
5. Domestic money stock	.03	7,715	7,316	.12	.14
6. Private-sector financial assets	.03	9,548	9,133	.12	.14
7. Manufacturing output	.03	18	19	.05	.05
8. Public authority financial deficit	.13	12,009	12,126	.06	.20
9. Unemployment	.08	41	40	.09	.09
10. Wage-rate index	.16	4	4	.08	.01
11. Imports	.19	2,560	1,905	.22	.19
12. Exports	.18	2,669	2,827	.15	.07
13. Consumers' expenditure	.04	8	9	.04	.04
14. Gross investment	.12	1,449	1,697	.08	.02
Monthly series					
1. Net foreign assets	.30	3,846	2,803	.25	.20
2. Public authority floating debt	.22	19,684	22,667	.16	.18
3. Net money-creating operations	.30	7,006	6,508	.20	.12
4. Miscellaneous items	.11	6,837	7,490	.19	.20
5. Domestic money stock	.07	26,011	23,842	.13	.12
6. Manufacturing output	.07	72	74	.06	.14
7. Male unemployment	.04	135,260	136,282	.04	.05
8. Industrial unemployment	.36	110,787	136,009	.20	.17
9. Unemployment construction industry	.19	107,252	135,624	.30	.31

NOTE: For the formulas of the measures of this table, see the Appendix.

with Census X-11 than with the STM method. Apparently, the recent shift in the seasonal component of this series is difficult to determine. In the case of the quarterly series of net foreign assets and of imports, especially at the end of the observation period, the seasonal component of Census X-11 is much more pronounced than that of the STM method. A remarkable difference between both adjustment methods shows up for the quarterly series of gross investment. According to the STM method, the seasonal component remains virtually stable over the whole observation period, whereas according to Census X-11, the values of the component in the first and second quarters shift to a considerable extent. Thus, at the end of the observation period, both methods yield nearly identical seasonal components, but not at the beginning.

Measured by the inequality coefficients, the largest difference in the case of the monthly series occurs with industrial unemployment. This difference becomes apparent mainly at the end of the observation period. The STM method yields much greater seasonal fluctuations for this series than Census X-11. For net money-creating operations too, the difference between the two adjustment methods shows up mainly at the end of the observation period. In the case of net foreign assets, the difference is spread over the whole observation period. The two methods are remarkably close to being identical in determining seasonality for the monthly series of male unemployment.

The mean absolute values of the seasonal components in Table 5 show that neither method systematically yields the largest seasonal component and hence that the size of the seasonal component does not depend on the adjustment method used. Table 5 also gives the mean absolute year-to-year change in the seasonal pattern in relation to the annual sum of the absolute value of the seasonal component as a measure of the extent to which the adjustment method tends to identify changes in the seasonal pattern. The results show no systematic differences between the seasonal adjustment methods with respect to flexibility either.

5. CONCLUSIONS

This article describes the use of structural time series models for seasonal adjustment. Application of the STM method to several Dutch macroeconomic time series proves it to be an attractive alternative to the Census X-11 method. A comparison of the two methods by means of formal criteria used in earlier comparative analyses shows that, according to most of these criteria, the two methods match each other closely. As to idempotency, the STM method scores much better than Census X-11, since it is virtually fully idempotent. The stability with respect to the whole observation period is slightly better with Census X-11 than that with the STM method. The reason is that, when new data are added, in the case of Census X-11 the seasonal adjustment of figures dating back four years or more by definition does not change any more, whereas when using the STM method the addition works through on all observations. As to the most recent data, stability proves slightly better when using the STM method than when using Census X-11.

The great advantage of the STM method over Census X-11, however, is the model-based approach (see Bell and Hillmer 1984). In the STM method, the decomposition of the series into the three components is based on a model whose characteristics differ for each series. This enables interpretation of the seasonal adjustment in terms of those characteristics. The use of structural time series models makes the seasonal adjustment even more interpretable than when it is based on ARIMA models [such as MSX; see Burman (1980)]. Moreover, it can be shown theoretically that, in its reduced form, Census X-11 is (almost) a special case of one of the models considered in the STM method, so for those series in which Census X-11 yields an appropriate specification, both methods are equivalent.

In this article, we have examined only two alternative models. Expansion of this number most probably will ameliorate the performance of the STM method. Moreover, the flexibility of the method will become greater if estimation techniques that determine a greater number of parameters by estimation, such as the autocorrelation parameter in the specification of the seasonal component, are used. Besides, the identification procedure and the selection of the most suitable model need more sophistication, since in our study the preliminarily identified specification sometimes had to be replaced by the other one. Such a sophistication would certainly pay off, since the results from our preliminary version of the STM method already seriously challenge the use of Census X-11 at statistical agencies from the practitioner's point of view.

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APPENDIX: SEASONAL ADJUSTMENT CRITERIA

Symbols

O—Original series.

S—Seasonal component.

SA—Seasonally adjusted series. $\overline{SA}$ means average value of SA over the estimation period.

S^*—Seasonal component in seasonally adjusted series.

t—Time index.
qm—Four for quarterly series, 12 for monthly series.
n—Number of observations.
r_k—kth autocorrelation coefficient of the irregular component.
ly—Latest year of the seasonally adjusted period.
n_y—Number of observations up to and including the year y.
S^y—Seasonal component obtained by seasonal adjustment up to and including the year y.
n^*—Number of full calendar years in the estimation period.

Formal Criteria (Table 4)

(a) Mean absolute percentage change:

$$\frac{100}{n-1} \sum_{t=2}^{n} \left| \frac{SA_t - SA_{t-1}}{SA_{t-1}} \right| . \tag{A.1}$$

(b) Orthogonality:

$$\frac{\sum_{t=1}^{n} (S_t - \bar{S})(SA_t - \overline{SA})}{(\sum_{t=1}^{n} (S_t - \bar{S})^2 \sum_{t=1}^{n} (SA_t - \overline{SA})^2)^{1/2}} . \tag{A.2}$$

(c) Idempotency:

$$\frac{1}{n} \sum_{t=1}^{n} 100 \left| \frac{S_t^*}{SA_t} \right| . \tag{A.3}$$

(d) Stability (common years):

$$\tfrac{1}{2}(ST_{ly}^{ly-1} + ST_{ly-1}^{ly-2}) \tag{A.4}$$

when

$$ST_y^{y-1} = \frac{1}{n_{y-1}} \sum_{t=1}^{n_{y-1}} 100 \left| \frac{S_t^y - S_t^{y-1}}{O_t} \right| .$$

Stability (latest years):

$$\tfrac{1}{2}(STL_{ly}^{ly-1} + STL_{ly-1}^{ly-2}) \tag{A.5}$$

when

$$STL_y^{y-1} = \frac{1}{qm} \sum_{t=n_{y-2}+1}^{n_{y-1}} 100 \left| \frac{S_t^y - S_t^{y-1}}{O_t} \right|$$

Comparison Criteria (Table 5)

(a) Theil's inequality coefficient:

$$\frac{(\sum_{t=1}^{n} (SA_t^{(1)} - SA_t^{(2)})^2)^{1/2}}{(\sum_{t=1}^{n} SA_t^{(1)2})^{1/2} + (\sum_{t=1}^{n} SA_t^{(2)2})^{1/2}} . \tag{A.6}$$

(b) Mean sum of the absolute values of the seasonal component in one year:

$$\frac{1}{n^*} \sum_{t=n_1+1}^{n_{ly}} |S_t| . \tag{A.7}$$

(c) Mean absolute change:

$$\frac{1}{n^*-1} \sum_{j=1}^{n^*-1} \frac{\sum_{i=1}^{s} |S_{j*s+i} - S_{(j-1)*s+i}|}{\sum_{i=1}^{s} |S_{j*s+i}|} . \tag{A.8}$$

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